Technical Comments

Comment on "A Paradoxical Case in a Stability Analysis"

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PARNES[1] has analyzed a specific beam buckling problem and discovered a paradox in that, for given conditions, the buckling load may increase with an increase in the overall length of the structural member. The author has not checked these analyses but, by considering a similar but simpler model, is able to show that the "paradox" is not unique and may be imaginary.

Two vertical bars AB and BC are initially in line and pinned at the points A, B, and C. A compressive vertical load P acts at the point C, which may move only vertically, and a lateral spring of stiffness β acts at the common pinned joint B (Fig. 1).

This model is a simple modification of those used by Timoshenko and Gere[2] to demonstrate the energy method in buckling analyses and is a simpler form of that used by Parnes,[1] with the rigid member AB and the lateral spring replacing his elastic member AB. For a small angular displacement α of AB, the lateral displacement of the spring is αL, and the downward vertical movement of point B is $\alpha^2 L/2$.

The overall downward vertical movement of point C is therefore

$$\delta = \tfrac{1}{2}\alpha^2 L[1+(1/c)] \tag{1}$$

and equating the strain energy in the spring to the work done by P gives

$$P(\alpha^2 L/2)[1+(1/c)] = \tfrac{1}{2}\beta(\alpha L)^2 \tag{2}$$

Therefore, the critical value of P is given by

$$P = \beta L/[1+(1/c)] \tag{3}$$

and, as c increases from zero through unity to infinity, one obtains the results shown in Table 1. Thus as c increases, i.e., overall length of the structure increases, the critical load also increases.

This is not a paradox, however, as can be shown easily by reconsidering the basic physical model. When c is infinite, the applied load P is transmitted to member AB as a vertical load P at B, and the critical load is $P/\beta L = 1$, as in Ref. 2. As c decreases to, say, 1, the equivalent force in member CB, acting at B, for a small lateral displacement of B, must be greater than P even though the vertical component is still P, and this results in a decrease in critical load P. Examination of Eq. (1) also shows that the downward movement of point C increases significantly as c decreases, resulting in a corresponding decrease in the critical load.

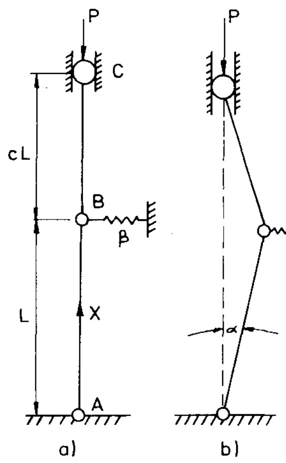

Fig. 1 Geometry of problem.

Table 1 Relationship of *c* to critical load

c	0	0.5	1	2	3	4	α
$P/\beta L$	0	0.3	0.5	0.6	0.75	0.8	1

The limiting condition at $c \rightarrow 0$, giving $P/\beta L \rightarrow 0$, is subject to the proviso that, if member BC exists and has zero length ($c = 0$), then there can be no instability, as point B of member AB will be constrained from moving laterally. Thus $P_{crit} \rightarrow \alpha$ and is not zero. This apparent paradox is thus explained.

References

[1] Parnes, R., "A Paradoxical Case in a Stability Analysis," *AIAA Journal*, Vol. 15, Oct. 1977, pp. 1533-1534.

[2] Timoshenko, S. and Gere, J., *Theory of Elastic Stability*, McGraw-Hill, New York, 1961, pp. 83-86.

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Reply by Author to D.J. Johns

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THE author notes that Dr. Johns, in considering a simple system consisting of rigid bars, has presented another model for which increased stability is obtained with increased length of a member. Certainly, no claim was made by the author that the model considered in the original paper represented a unique case.

The physical reason for which this phenomenon occurs in the two cases cited is explained by Dr. Johns in terms of an equivalent force. A more precise physical explanation may, perhaps, be given as follows. In both models, member BC can transmit only a force concurrent with the chord BC of the deflected position. Consequently, for any given small

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